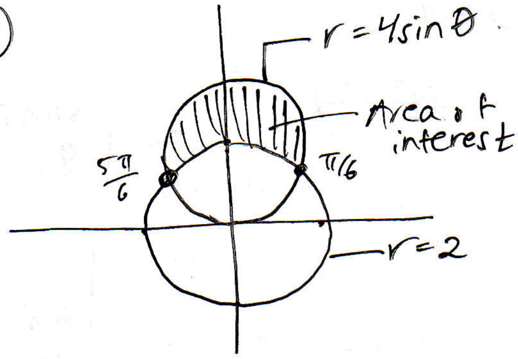
PASS MOCK TEST 3 - ANSWER KEY

(1)



Set up integral : $\int_a^b \frac{1}{2} f(\theta)^2 \, d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4\sin\theta)^2 - (2)^2 \, d\theta = \boxed{\frac{4\pi}{3} + 2\sqrt{3}}$

(2) a) Ratio Test $\Rightarrow$ b/c of ab. values this series is absolutely $\boxed{\text{convergent}}$ since $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$

b) Take a limit $\Rightarrow$ b/c this is a sequence not a series; divide a_n by "n" first then apply limit as $n \to \infty$

$\lim_{n\to\infty} a_n = -1$ which is not $\emptyset$ $\therefore$ it is $\boxed{\text{convergent}}$

c) Alt series test - convergent

i) $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$ $\therefore$ decreasing

ii) $\lim_{n\to\infty} a_n = \emptyset$

BUT! By p-test - divergent

$\frac{1}{\sqrt{n}}$ is a divergent series since $p = 1/2$

$\therefore$ series is $\boxed{\text{conditionally convergent}}$

d) Comparison test & p-test $\Rightarrow$ change series to have a base of "n" by raising it to the power of e and rearranging the compare it to p-series w/ $p=2$

$$\frac{1}{n^{\ln(\ln n)}} \leq \frac{1}{n^2}$$ for "n" that is substantially large

$\therefore$ B/c $\frac{1}{n^2}$ is convergent, the series that is smaller must also $\boxed{\text{converge.}}$

③ Logically you would pay Cindy \$1 b/c you'd atleast break even. The question is; if you neglect the 1st coin flip and make it to the second coin flip, how much would you wager. set up chart

outcome	probability	Pay off	Expect profit
H	1/2	\$1	$1/2 \cdot 1 =$ \$0.50
T H	1/4	\$2	$1/4 \cdot 2 =$ \$0.50
T T H	1/8	\$3	$1/8 \cdot 3 =$ \$0.375
T T T H	1/16	\$4	$1/16 \cdot 4 =$ \$0.25
T T . . . H	$1/2^n$	\$n	$1/2^n \cdot n = \$n/2^n$

sum is \$1

Sum is \$1

Geometric series w/ $r = \left(\frac{1}{2}\right)^n$ ∴ summing up the terms ignoring the first 2 terms gives you $\sum_{n=3}^{\infty} \left(\frac{1}{2}\right)^n \Rightarrow S_n = \frac{a}{1-r} = \frac{1/2}{1-1/2} = 1$

sum of 1st 2 terms

sum of terms n=3 to ∞.

∴ $1 + 1 = \boxed{\$2}$ ← ∴ You should wager less than \$2 to make a profit in the long run b/c after the first coin flip you will make atleast \$2

④ Ratio Test ⇒ when apply $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}|x-2|$ ★ there's also n terms missing here we are only looking at x-values here

a)

∴ Radius of con. is 2, interval of convergence is $\boxed{(0,4)}$ after testing the 2 endpoints.

b) knowing that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ ⇒ multiply by x^2 to obtain series of interest

use ratio test ⇒ $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = |x|$ ★ Note there are "n" terms missing we are only looking at x-values here

∴ Radius of convergence is equal to ∞ b/c x can range from $\boxed{(-\infty, \infty)}$

BONUS: $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ ⇒ converges if compared to $\frac{1}{(n-1)^2}$

↓

convergent p-series

DO NOT COMPARE TO $\frac{1}{n^2}$
IT WILL NOT WORK!