

PASS MATH 1014 MOCK TEST 3 WINTER 2016

Weekly sessions held on
MONDAY 6 PM LSB 107
WEDNESDAY 6PM LSB 107
PASS Leader: Cindy Tran

Please note: Do not substitute this mock test for studying. The questions included are only examples of what may appear on the midterm and are not necessarily representative of the degree of difficulty.

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PASS MATH 1014 Winter 2016

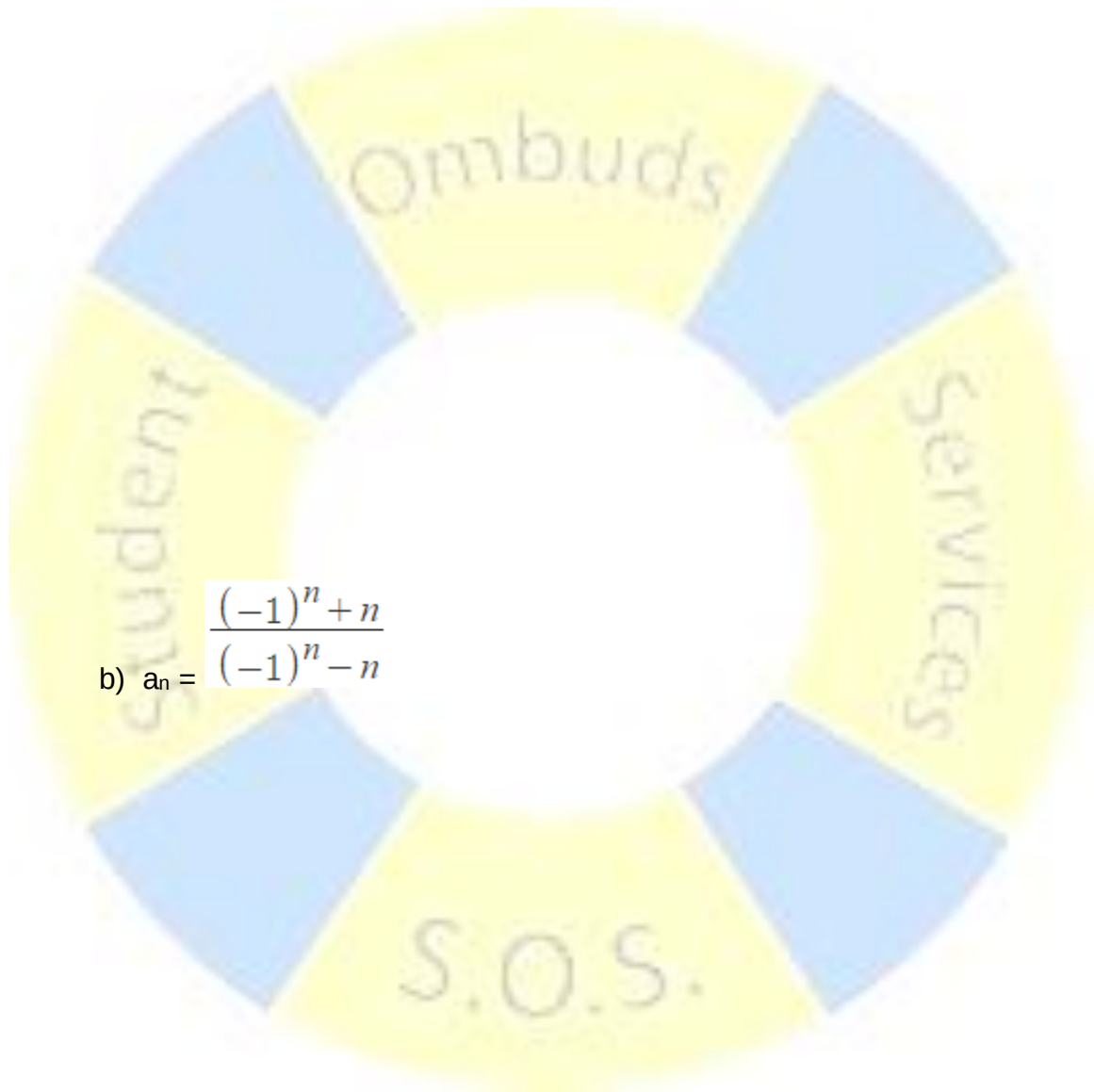
1. Find the area between the graphs $r = 2$ and $r = 4 \sin \theta$. Provide a sketch of the overlapping regions.



2. Determine whether the following are convergent (absolutely or conditionally) or divergent and state how you determined this.

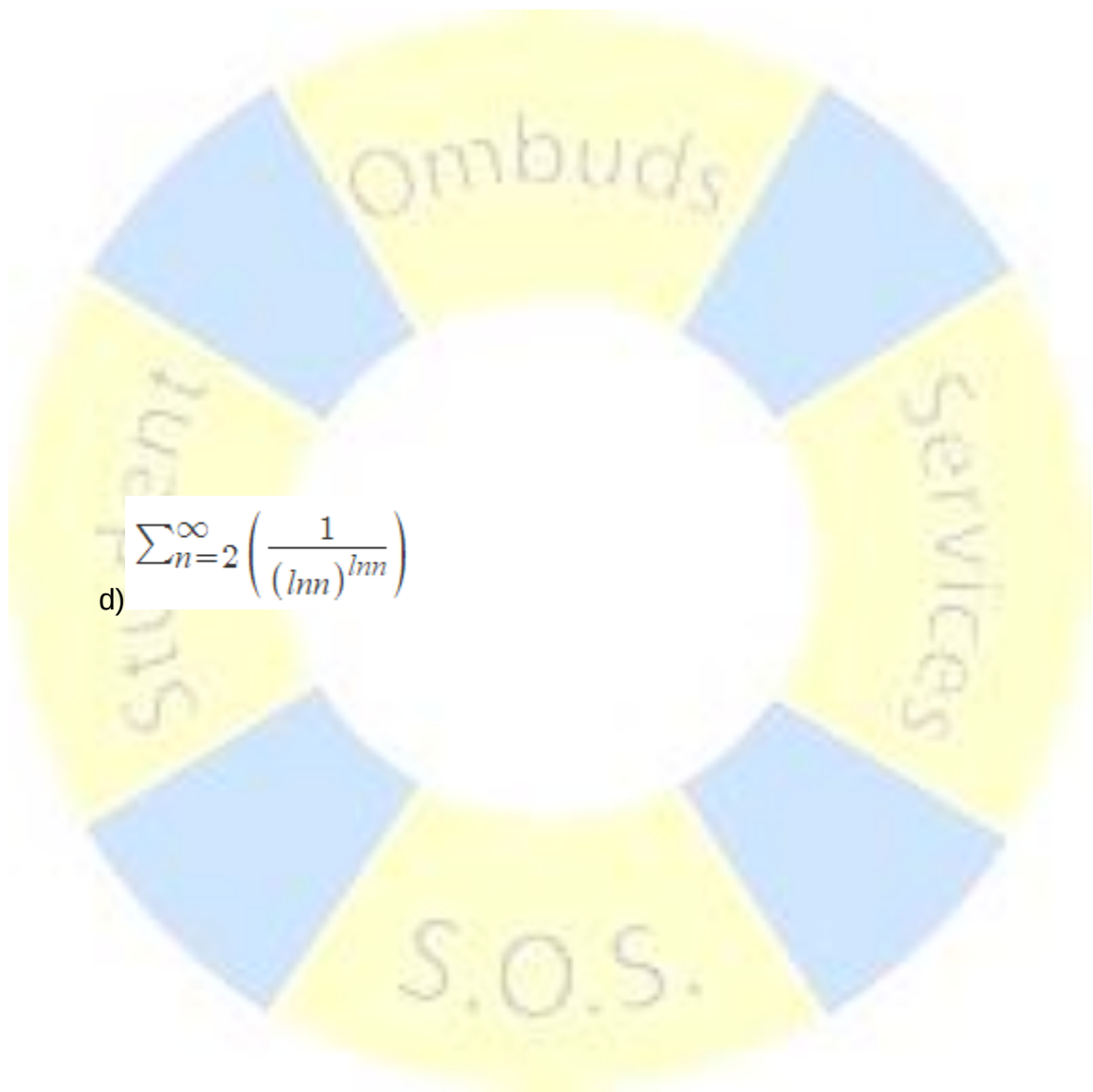
a) $\sum_{n=1}^{\infty} \frac{2^n n!}{(2n)!}$

b) $a_n = \frac{(-1)^{n+n}}{(-1)^{n-n}}$



c) $\sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{\sqrt{n}} \right)$

d) $\sum_{n=2}^{\infty} \left(\frac{1}{(lmn)^{lmn}} \right)$



3. In PASS, Cindy suggests to play a game with you to encourage studying series. She has a coin with two different sides, heads and tails, which she will toss. Each toss she throws will have an EQUAL chance of landing on either heads or tails. Here's the fun part. If it lands on heads, she will give you \$1, if it lands on tails, she will toss the coin again. If on the second toss, it is heads, you get \$2, and if it is tails she will toss the coin again. On the third toss, if it is heads, \$3 is yours, and if tails she will toss it again... You get the pattern by now. As a strategic player, how much should you be willing to wager Cindy to play this game and make a profit?



4. a) Find the interval and radius of convergence for the following series

$$\sum_{n=0}^{\infty} \frac{n^3}{2^n} (x-2)^n$$

- b) Find a power series representation for e^{x^2} and using this, find the radius of convergence for your series



SCRAP PAPER:

BONUS QUESTION:

$$\sum_{n=2}^{\infty} \left(\frac{1}{n^2 - 1} \right)$$

Prove that the series converges.



There's flow chart I found online that maybe useful to you when studying on the next page. Best of luck on the Actual Test! ☺

SERIES CONVERGENCE/DIVERGENCE FLOW CHART

TEST FOR DIVERGENCE

Does $\lim_{n \rightarrow \infty} a_n = 0$?

NO

$\sum a_n$ Diverges

YES

p-SERIES

Does $a_n = 1/n^p, n \geq 1$?

YES

Is $p > 1$?

YES

$\sum a_n$ Converges

NO

$\sum a_n$ Diverges

NO

GEOMETRIC SERIES

Does $a_n = ar^{n-1}, n \geq 1$?

YES

Is $|r| < 1$?

YES

$\sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$

NO

$\sum a_n$ Diverges

NO

ALTERNATING SERIES

Does $a_n = (-1)^n b_n$ or $a_n = (-1)^{n-1} b_n, b_n \geq 0$?

YES

Is $b_{n+1} \leq b_n$ & $\lim_{n \rightarrow \infty} b_n = 0$?

YES

$\sum a_n$ Converges

NO

TELESCOPING SERIES

Do subsequent terms cancel out previous terms in the sum? May have to use partial fractions, properties of logarithms, etc. to put into appropriate form.

YES

Does $\lim_{n \rightarrow \infty} s_n = s$ finite?

YES

$\sum a_n = s$

NO

$\sum a_n$ Diverges

NO

TAYLOR SERIES

Does $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$?

YES

Is x in interval of convergence?

YES

$\sum_{n=0}^{\infty} a_n = f(x)$

NO

$\sum a_n$ Diverges

NO

Try one or more of the following tests:

COMPARISON TEST

Pick $\{b_n\}$. Does $\sum b_n$ converge?

YES

Is $0 \leq a_n \leq b_n$?

YES

$\sum a_n$ Converges

NO

NO

Is $0 \leq b_n \leq a_n$?

YES

$\sum a_n$ Diverges

LIMIT COMPARISON TEST

Pick $\{b_n\}$. Does $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$ finite & $a_n, b_n > 0$?

YES

Does $\sum_{n=1}^{\infty} b_n$ converge?

YES

$\sum a_n$ Converges

NO

$\sum a_n$ Diverges

INTEGRAL TEST

Does $a_n = f(n)$, $f(x)$ is continuous, positive & decreasing on $[a, \infty)$?

YES

Does $\int_a^{\infty} f(x) dx$ converge?

YES

$\sum_{n=a}^{\infty} a_n$ Converges

NO

$\sum a_n$ Diverges

RATIO TEST

Is $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| \neq 1$?

YES

Is $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$?

YES

$\sum a_n$ Abs. Conv.

NO

$\sum a_n$ Diverges

ROOT TEST

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$?

YES

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$?

YES

$\sum a_n$ Abs. Conv.

NO

$\sum a_n$ Diverges