Solutions Test 2 ①

1(a) $$y = \frac{2x}{x^2+1}, \quad (1,1)$$

$$\left(\frac{2x}{x^2+1}\right)' = \frac{(2x)'(x^2+1) - 2x(x^2+1)'}{(x^2+1)^2} =$$

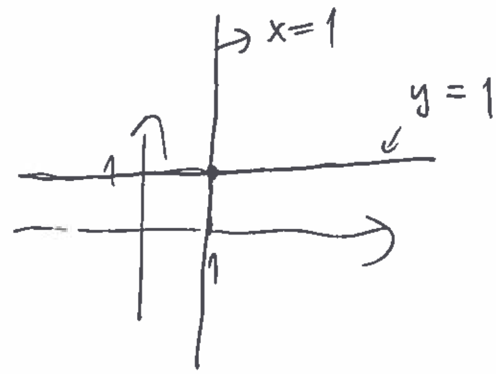
$$= \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$$= \frac{2(1-x^2)}{(x^2+1)^2} \Rightarrow \frac{2(1-1^2)}{(1^2+1)^2} = 0$$

• the tangent line at $(1,1)$:

$$y = 0\cdot(x-1) + 1$$

$$\Rightarrow \underline{y = 1}$$



• the normal line at $(1,1)$: $\underline{x = 1}$

(1b) $$f(x) = \sqrt{x}\, e^x = x^{\frac{1}{2}} e^x$$

$$\left(x^{\frac{1}{2}} e^x\right)' = \left(x^{\frac{1}{2}}\right)' e^x + x^{\frac{1}{2}} \left(e^x\right)' =$$

$$= \frac{1}{2} x^{\frac{1}{2}-1} e^x + x^{\frac{1}{2}} e^x = \frac{1}{2} x^{-\frac{1}{2}} e^x + x^{\frac{1}{2}} e^x \quad //$$

$$f''(x) = \left(\frac{1}{2}x^{-\frac{1}{2}}e^{x} + x^{\frac{1}{2}}e^{x}\right)'$$

$$= \left(\frac{1}{2}x^{-\frac{1}{2}}e^{x}\right)' + \left(x^{\frac{1}{2}}e^{x}\right)'$$

$$= \frac{1}{2}\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}e^{x} + \frac{1}{2}x^{-\frac{1}{2}}e^{x} + \frac{1}{2}x^{-\frac{1}{2}}e^{x} + x^{\frac{1}{2}}e^{x}$$

$$= -\frac{1}{4}x^{-\frac{3}{2}}e^{x} + x^{-\frac{1}{2}}e^{x} + x^{\frac{1}{2}}e^{x} \;//$$

(2a) $$\left(e^{\frac{x}{x-1}}\right)' = e^{\frac{x}{x-1}}\cdot\left(\frac{x}{x-1}\right)' = e^{\frac{x}{x-1}}\cdot\frac{(x-1)-x}{(x-1)^{2}}$$

$$= e^{\frac{x}{x-1}}\cdot\frac{-1}{(x-1)^{2}}$$

2b) $$\left(\sin^{2}\left(e^{\sin^{2}x}\right)\right)' = \left(\sin\left(e^{\sin^{2}x}\right)\cdot\sin\left(e^{\sin^{2}x}\right)\right)'$$

$$= \left(\sin\left(e^{\sin^{2}x}\right)\right)'\sin\left(e^{\sin^{2}x}\right) + \sin\left(e^{\sin^{2}x}\right)\cdot\left(\sin\left(e^{\sin^{2}x}\right)\right)'$$

The Product Rule

$$= 2\cdot\sin\left(e^{\sin^{2}x}\right)\left(\sin\left(e^{\sin^{2}x}\right)\right)'$$

$$= 2\sin\left(e^{\sin^{2}x}\right)\cdot\cos\left(e^{\sin^{2}x}\right)\cdot\left(e^{\sin^{2}x}\right)'$$

$$= 2\sin\left(e^{\sin^{2}x}\right)\cdot\cos\left(e^{\sin^{2}x}\right)\cdot e^{\sin^{2}x}\cdot\left(\sin^{2}x\right)'$$

$$= \sin\left(2\cdot e^{\sin^{2}x}\right)\cdot e^{\sin^{2}x}\,2\sin x\cdot\cos x$$

$$= e^{\sin^2 x} \sin\left(2 e^{\sin^2 x}\right) \cdot \sin(2x) \quad //$$

$$\cancel{(2c)\ \left(\tan 4\sqrt{x} + \pi\right)' = \left(\tan 4\sqrt{x}\right)' + 0}$$

(2c)

$$\left(\tan^{-1}\sqrt{x} + \pi\right)' = \left(\tan^{-1}\sqrt{x}\right)' + 0$$

$$= \frac{1}{1 + (\sqrt{x})^2} \cdot (\sqrt{x})' = \frac{1}{1+x} \cdot \frac{1}{2} x^{-\frac{1}{2}} \quad //$$

(3a)

$r(t)$ = the radius at time t

$V(t)$ = the volume at time t

$$r'(t) = 4 \ mm/s \quad \forall t \geq 0$$

- $V(t) = \frac{4}{3}\pi r^3(t) \quad \Big/ \frac{d}{dt}$

$$V'(t) = \left(\frac{4}{3}\pi r^3(t)\right)' = \frac{4}{3}\pi\left(r^3(t)\right)' = \frac{4}{3}\pi \cdot 3 r^2(t)\, r'(t)$$

$$= 4\pi r^2(t) \cdot r'(t)$$

$$= 4\pi r^2(t) \cdot 4$$

$$= 16\pi r^2(t)$$

if $d = 80$ mm $\Rightarrow r = \frac{d}{2} = 40$ mm

$$\Rightarrow V'(t) = 16 \cdot \pi \cdot (40)^2 \ mm^3/s. \quad //$$

(3b)

$$x \cdot e^{y(x)} = x - y(x) \quad / \frac{d}{dx}$$

$$\left(x e^{y(x)}\right)' = \left(x - y(x)\right)'$$

$$1 \cdot e^{y(x)} + x \cdot \left(e^{y(x)}\right)' = 1 - y'(x)$$

$$e^{y(x)} + x e^{y(x)} y'(x) = 1 - y'(x)$$

$$e^{y} + x e^{y} y' = 1 - y'$$

$$\Rightarrow \quad y'\left(1 + x e^{y}\right) = 1 - e^{y}$$

$$y' = \frac{1 - e^{y}}{1 + x e^{y}}$$