Solutions Test 1.

1(a) $f(x) = \dfrac{(x^2+1)^2}{3x-2x^2}$

$3x - 2x^2 = 0$

$x(3-2x) = 0$

$x_1 = 0$

$x_2 = \frac{3}{2}$

$$\lim_{x\to 0^+} \frac{\overbrace{(x^2+1)^2}^{>0}}{\underbrace{x}_{>0}\underbrace{(3-2x)}_{>0}} = \infty$$

or

$$\lim_{x\to 0^-} \frac{\overbrace{(x^2+1)^2}^{>0}}{\underbrace{x}_{<0}\underbrace{(3-2x)}_{>0}} = -\infty$$

$\Rightarrow$

$\Rightarrow$ $x=0$ is a vertical asymptote.

$$\lim_{x\to \frac{3}{2}^+} \frac{(x^2+1)^2}{x(3-2x)} = -\infty$$

or

$$\lim_{x\to \frac{3}{2}^-} \frac{(x^2+1)^2}{x(3-2x)} = \infty$$

$\Rightarrow$ $x = \frac{3}{2}$ is a vertical asymptote

1(b) $$\lim_{x\to\infty} \frac{2e^x}{e^x-6} = \lim_{x\to\infty} \frac{2}{1-\frac{6}{e^x}} = \frac{2}{1-0} = 2$$

$\Rightarrow$ $y=2$ is a horizontal asymptote

$$\lim_{x \to -\infty} \frac{2e^x}{e^x - 6} = \frac{2 \cdot 0}{0-6} = \frac{0}{-6} = 0$$ 2

$\Rightarrow y = 0$ is a horizontal asymptote

2(a) $$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2+x} \right) = \lim_{x \to 0} \frac{x^2}{x(x^2+x)}$$

$$= \lim_{x \to 0} \frac{x^2}{x^2(1+x)} = \lim_{x \to 0} \frac{1}{1+x} = 1 //$$

2(b) $$\lim_{x \to 2} \frac{\sqrt{4x+1} - 3}{x-2} = \lim_{x \to 2} \frac{\sqrt{4x+1} - 3}{x-2} \cdot \frac{\sqrt{4x+1} + 3}{\sqrt{4x+1} + 3}$$

$$= \lim_{x \to 2} \frac{4x+1-9}{(x-2)(\sqrt{4x+1}+3)}$$

$$= \lim_{x \to 2} \frac{4(x+2)}{(x-2)(\sqrt{4x+1}+3)} = \lim_{x \to 2} \frac{4}{\sqrt{4x+1}+3}$$

$$= \frac{4}{\sqrt{4 \cdot 2+1}+3} = \frac{4}{3+3} = \frac{4}{6} = \frac{2}{3} //$$

2(c) $0 \le \cos^2 \frac{\pi}{x} \le 1, \ \forall x \sim 0, x \neq 0$

$$\Rightarrow \underbrace{0 \cdot \sqrt{x^3+x^2}}_{\downarrow 0} \le \sqrt{x^3+x^2} \cos^2 \frac{\pi}{x} \le \underbrace{\sqrt{x^3+x^2}}_{\downarrow 0} \overset{\text{the Squeeze Thm}}{\Longrightarrow} \lim_{x \to 0} \sqrt{x^3+x^2} \cos^2 \frac{\pi}{x} = 0 //$$

(3a) if $x<\frac{\pi}{4} \Rightarrow f(x)=\sin x$ is continuous

becase $\sin x$ is continuous

if $x>\frac{\pi}{4} \Rightarrow f(x)=\cos x$ is continuous

becase $\cos x$ is continuous

• $x=\frac{\pi}{4}$ $\quad \lim\limits_{x\to\frac{\pi}{4}^-} f(x) = \lim\limits_{x\to\frac{\pi}{4}^-} \sin x =$

$$= \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\lim_{x\to\frac{\pi}{4}^+} f(x) = \lim_{x\to\frac{\pi}{4}^+} \cos x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}^-} = \lim_{x\to\frac{\pi}{4}^+} = \frac{\sqrt{2}}{2} \Rightarrow \lim_{x\to\frac{\pi}{4}} f(x) \text{ exists}$$

and $\lim\limits_{x\to\frac{\pi}{4}} f(x) = \frac{\sqrt{2}}{2}$.

On the other hand, $f\left(\frac{\pi}{4}\right) \overset{\frac{\pi}{4}\geq\frac{\pi}{4}}{=} \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\Rightarrow \lim\limits_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right) \Rightarrow f$ is continuous at $x=\frac{\pi}{4}$

35 $f(x) = \frac{x^2+x-12}{x+4} = \frac{x^2+4x-3x-12}{x+4} =$

$$= \frac{x(x+4)-3(x+4)}{x+4} = x-3$$

$\Rightarrow \lim_{x \to -4} f(x) = \lim_{x \to -4} (x-3) = -4-3 = -7$

$\Rightarrow$ we have to define $f(-4) = -7$

0